

# 13-2

## Probability Distributions and Frequency Tables

### Common Core State Standards

S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified . . . Also

S-CP.A.5

MP 1, MP 2, MP 3

**Objective** To make and use frequency tables and probability distributions



Think about how you can use data in tables to find probabilities.



**Getting Ready!**

The table at the right shows the speeds of cars as they pass a certain mile marker on highway 66. The speed limit is 65 mph. What is the total number of cars that passed the marker? What is the probability that a car stopped at random will be traveling faster than the speed limit?

Speed (mph)	Number of Cars
< 55	2
55–60	12
60–65	23
> 65	13

In the Solve It, you used information from the table to calculate probability. A **frequency table** is a data display that shows how often an item appears in a category.

**Essential Understanding** You can use data organized in tables that show frequencies to find probabilities.



### Lesson Vocabulary

- frequency table
- relative frequency
- probability distribution

take note

### Key Concept Relative Frequency

**Relative frequency** is the ratio of the frequency of the category to the total frequency.



### Problem 1 Finding Relative Frequencies

**Surveys** The results of a survey of students' music preferences are organized in this frequency table. What is the relative frequency of preference for rock music?

Use the frequency table to find the number of times rock music is chosen as the preference, and the total number of survey results.

$$\begin{aligned} \text{relative frequency} &= \frac{\text{frequency of rock music preference}}{\text{total frequency}} \\ &= \frac{10}{10 + 7 + 8 + 5 + 6 + 4} = \frac{10}{40} = \frac{1}{4} \end{aligned}$$

The relative frequency of preference for rock music is  $\frac{1}{4}$ .

Type of Music Preferred	Frequency
Rock	10
Hip Hop	7
Country	8
Classical	5
Alternative	6
Other	4

### Plan

How do you find the denominator for a relative frequency?

Find the sum of the frequencies in the frequency table.



- Got It?** 1. What is the relative frequency of preference for each type of music?
- classical
  - hip hop
  - country
  - Critical Thinking** Without calculating the rest of the relative frequencies, what is the sum of the relative frequencies for all the types of music? Explain how you know.

You can use relative frequency to approximate the probabilities of events.



### Problem 2 Calculating Probability by Using Relative Frequencies

A student conducts a probability experiment by tossing 3 coins one after the other. Using the results below, what is the probability that exactly two heads will occur in the next three tosses?

Coin Toss Result	HHH	HHT	HTT	HTH	THH	THT	TTT	TTH
Frequency	5	7	9	6	2	9	10	2

**Step 1** Find the number of times a trial results in exactly two heads.

The possible results that show exactly two heads are HHT, HTH, and THH.

The frequency of these results is  $7 + 6 + 2 = 15$ .

**Step 2** Find the total of all the frequencies.

$$5 + 7 + 9 + 6 + 2 + 9 + 10 + 2 = 50$$

**Step 3** Find the relative frequency of a trial with exactly two heads.

$$\text{relative frequency} = \frac{\text{frequency of exactly two heads}}{\text{total of the frequencies}} = \frac{15}{50} = \frac{3}{10}$$

Based on the data collected, the probability that the next toss will be exactly two heads is  $\frac{3}{10}$ .

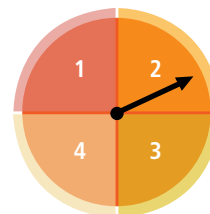
### Think

How can you use the frequency table to find the probability? The relative frequency is an approximation of the overall probability of the result.



- Got It?** 2. A student conducts a probability experiment by spinning the spinner shown. Using the results in the frequency table, what is the probability of the spinner pointing at 4 on the next spin?

Spinner Result	1	2	3	4
Frequency	29	32	21	18



A **probability distribution** shows the probability of each possible outcome. A probability distribution can be shown in a frequency table.



### Problem 3 Finding a Probability Distribution

**Archery** In a recent competition, 50 archers shot 6 arrows each at a target. Three archers hit no bull's eyes; 5 hit one bull's eye; 7 hit two bull's eyes; 7 hit three bull's eyes; 11 hit four bull's eyes; 10 hit five bull's eyes; and 7 hit six bull's eyes. What is the probability distribution for the number of bull's eyes each archer hit?

#### Know

The possible outcomes and the frequency of each outcome.

#### Need

The probabilities of each outcome.

#### Plan

Make a frequency table and use relative frequencies to complete the probability distribution.

First, create a frequency table showing all of the possible outcomes: 0, 1, 2, 3, 4, 5, or 6 bull's eyes and the frequency for each.

Next, use the table to find the relative frequencies for each number of bull's eyes. The relative frequencies are the probability distribution.

Probability Distribution of Bull's Eyes Hits							
Number of Bull's Eyes Hit	0	1	2	3	4	5	6
Frequency	3	5	7	7	11	10	7
Probability	$\frac{3}{50}$	$\frac{5}{50}$	$\frac{7}{50}$	$\frac{7}{50}$	$\frac{11}{50}$	$\frac{10}{50}$	$\frac{7}{50}$



**Got It?** 3. On a math test, there were 10 scores between 90 and 100, 12 scores between 80 and 89, 15 scores between 70 and 79, 8 scores between 60 and 69, and 2 scores below 60. What is the probability distribution for the test scores?



## Lesson Check

### Do you know HOW?

**Consumer Research** The results of a survey show the frequency of responses for preferred music formats. What are the relative frequencies of the following formats?

1. CD
2. MP3
3. Blu-ray
4. Radio
5. What is the probability distribution for the music formats?

Preferred Format	Frequency
CD	54
Radio	50
Blu-ray	10
MP3	28

### Do you UNDERSTAND?



6. Describe the sample space of possible outcomes for tossing a coin once.
7. **Error Analysis** A friend says that her sweepstakes ticket will either win or not win, so the theoretical probability of it winning must be 50%. Explain your friend's error.



## Practice and Problem-Solving Exercises



### A Practice

**Blood Drive** The honor society at a local high school sponsors a blood drive. High school juniors and seniors who weigh over 110 pounds may donate. The table at the right indicates the frequency of each donor blood type.

← See Problem 1.

**Blood Drive Results**

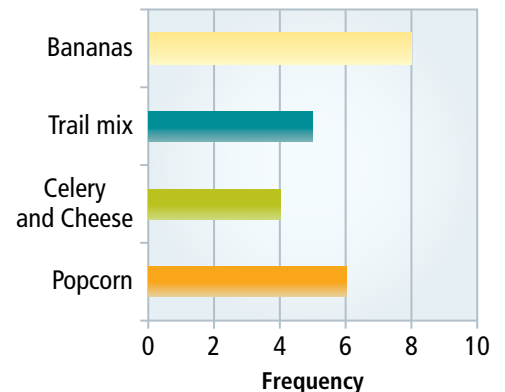
Blood Type	Frequency
O	30
A	25
B	6
AB	2

8. What is the relative frequency of blood type AB?
9. What is the relative frequency of blood type A?
10. Which blood type has the highest relative frequency? What is the relative frequency for this blood type?
11. The blood drive is extended for a second day, and the frequency doubles for each blood type. Does the relative frequency for each blood type change? Explain.
12. The data collected for new students enrolled at a community college show that 26 are under the age of eighteen, 395 are between the ages of eighteen and twenty-two, 253 are between the ages of twenty-three and twenty-seven, 139 are between the ages of twenty-eight and thirty-two, and 187 are over the age of thirty-two. What is the probability distribution for these data?

← See Problems 2 and 3.

Twenty-three preschoolers were asked what their favorite snacks are. The results are shown in the bar graph at the right.

**Favorite Snacks**



### B Apply

16. **Reasoning** The possible outcomes for a spinner are 1, 2, 3, 4, 5, and 6. The outcomes are equally likely. What is a probability distribution of the possible outcomes for one spin of the spinner?
17. **Text Messages** The table at the right shows numbers of text messages sent in one month by students at Metro High School.
  - a. If a student is chosen at random, what is the probability that the student sends 1500 or fewer text messages in one month?
  - b. If a student is chosen at random, what is the probability that the student sends more than 1500 messages in a month?

Number of Text Messages $t$	Number of Students
$t \leq 500$	25
$500 < t \leq 1500$	120
$1500 < t \leq 2500$	300
$t > 2500$	538

18. **Computers** The results of a survey of 80 households in Westville are shown at the right.

- a. What is the probability distribution of the number of computers in Westville households?
- b. If Westville has 15,200 households, predict the number of households that will have exactly 3 computers.
- c. How many households will have either two or three computers?

Computer Survey

Number of Computers	Frequency
0	12
1	29
2	31
3	6
More than 3	2

19. **Think About a Plan** You can make a probability distribution table for theoretical probabilities. What is a probability distribution for four tosses of a single coin?

- What are all of the possible outcomes?
- What is the probability of each outcome?
- How can you display this in a table?

20. **Employment** The table below shows the number of people (in thousands) working in each occupational category, according to the U.S. Bureau of Labor Statistics.

U.S. Occupational Categories of Employed Workers

Occupational Category	Management, professional, and related	Service	Sales and office	Natural resources, construction, maintenance	Production, transportation, and material moving
Number (thousands)	9773	2271	1456	289	177

- a. Make a table showing the relative frequency for each occupational category.
- b. If an employed person is randomly selected, what is the likelihood the person works in the Service category? Explain how you know.

21. a. Make a probability distribution for the sum of the faces after rolling two standard number cubes.  
b. Are the probabilities theoretical or based on experimental results? Explain.

22. A student chose 100 letters at random from a page of a textbook. Partial results are in the table at the right. Make a probability distribution for the data. Include a category for all other letters not represented by the table.

Letter	Tally
a	
e	
i	
n	
o	
r	
s	
t	

23. **Error Analysis** The results of a survey about students' favorite type of pet are shown in the probability distribution below. What is wrong with this probability distribution? What information do you need to correct the probability distribution?

Favorite Pet	Dogs	Cats	Birds	Fish	Hamsters
Probability	0.36	0.34	0.15	0.08	0.12

**Challenge**

24. A spinner has an equal number of even and odd numbers. The probability of getting an outcome of an even number on any spin is 50%. Make a probability distribution for even numbers for two spins, for three spins, and for four spins. How does the probability distribution change for each additional spin?

**Standardized Test Prep****SAT/ACT**

25. One hundred fifty students were asked the number of hours they spend on homework each night. Twenty students responded that they spend less than 1 hour on homework each night. What is the relative frequency of this response?
- (A) 0.13                      (B) 0.87                      (C) 20                      (D) 130
26. Which pair of angles are supplementary?
- (A)  $110^\circ$  and  $70^\circ$                       (B)  $25^\circ$  and  $65^\circ$   
(C)  $90^\circ$  and  $80^\circ$                       (D)  $160^\circ$  and  $200^\circ$
27. What is the maximum number of right angles possible in a pentagon?
- (A) 1                      (B) 2                      (C) 3                      (D) 4
28. What is the measure of an interior angle of a regular octagon?

**Short Response****Mixed Review**

Determine whether each of the following is an experimental or theoretical probability. Explain your choice.

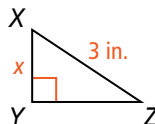
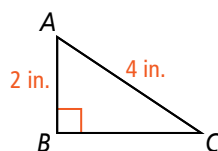
← See Lesson 13-1.

29. the probability of rolling two 5's when rolling two standard number cubes
30. the probability that a poll respondent chooses red as their favorite color
31. the probability of a number being even when randomly chosen from the whole numbers less than 11

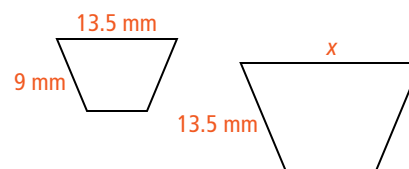
The pairs of figures are similar. Find  $x$ .

← See Lesson 7-2.

32.



33.



**Get Ready!** To Prepare for Lesson 13-3, do Exercises 34–36.

← See p. 891.

Simplify each ratio.

34.  $\frac{ab}{bc}$

35.  $\frac{24 \cdot 15}{8 \cdot 3}$

36.  $\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$